Number of Terms.	Mid-Term or Pair of Mid-Terms.	The Series.
1	15	15
3	5	$4+5+6$
5	3	$1+2+3+4+5$
15	1	$-6-5-4-3-2-1+0+1+2+3+4+5+6+7+8$
2	7,8	$7+8$
6	2,3	$0+1+2+3+4+5$
10	1,2	$-3-2-1+0+1+2+3+4+5+6$
30	0,1	$-14-13-\ .\ .\ .\ +0+1\ .\ .\ .\ +14+15$

NOTE ON PRIME NUMBERS.

By DERRICK N. LEHMER, University of California.

It is a well known theorem that it is possible to find an arbitrarily great number of consecutive composite numbers. This appears from the values which the expression $n!+r$ takes for $r=2, 3, \ldots, n$. This theorem furnishes an interesting proof of the theorem that the number of primes less than or equal to x is not determined by a function of x which is a polynomial in x of finite degree. For if $f(x)$ were such a function of degree n, then for $x=(n+2)!+r$, $f(x)$ must keep the same value for $r=2, 3, 4, \ldots, n+2$. If this value is k, then $f(x)-k=0$ is an equation of degree n with $n+1$ roots, which is impossible.

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

362. Proposed by JAMES F. LAWRENCE, Stillwater, Okla.

Show that the number of solutions in positive integers, zero included, of the equation $x+2y+3z=6n$, is $3n^2+3n+1$.

Solution by W. J. GREENSTREET, M. A., Editor of the Mathematical Gazette, Burghfield, England.

$x+2y+3z=6n$. z may have any value from 0 to $2n$, inclusive.

Hence we may assign to it any even or odd value from 0 to $2n$, inclusive.

i. Let $z=2r$. $x+2y=6(n-r)$. y may have any value from 0 to $3(n-r)$.

$\therefore$ There are $3(n-r)+1$ solutions when z is $2r$.

$\therefore$ Total number of solutions for z even is

$$\sum_{r=0}^{r=n}[3(n-r)+1]=(3n+1)(n+1)-\frac{3n.(n+1)}{2}$$

$$=\frac{n+1}{2}(6n+2-3n)=\frac{(n+1)(3n+2)}{2}.$$

ii. Let $z=2r+1$. $x+2y=6(n-r)-3$. y may have any value from 0 to $3(n-r)-2$.

$\therefore 3(n-r)-2+1=3(n-r)-1$ solutions.

$\therefore$ Total number of solutions when z is odd:

$$=\sum_{r=0}^{r=n-1}[3(n-r)-1]=(3n-1)n-\frac{3n(n-1)}{2}=\frac{n}{2}(6n-2-3n+3)=\frac{n}{2}(3n+1).$$

$\therefore$ The total number of solutions

$$=\frac{(n+1)(3n+2)}{2}+\frac{n(3n+1)}{2}=\frac{3n^2+5n+2+3n^2+n}{2}=3n^2+3n+1.$$

Also solved by H. Prime, J. Scheffer, H. C. Feemster, and A. M. Harding.

363. Proposed by E. B. ESCOTT, Ann Arbor, Mich.

(a) If a and n be positive integers, the integral part of $[a+\sqrt{(a^2-1)}]^n$ is odd.

(b) If a and n be positive integers, the integral part of $[\sqrt{(a^2+1)}+a]^n$ is odd when n is even and even when n is odd. [From Todhunter's *Algebra*, p. 353].

I. Solution by the PROPOSER.

Proof. (a) Let $[a+\sqrt{(a^2-1)}]^n=P+Q\sqrt{(a^2-1)}=m$.
Then $[a-\sqrt{(a^2-1)}]^n=P-Q\sqrt{(a^2-1)}=1/[a+\sqrt{(a^2-1)}]^n$.
$\therefore 0<P-Q\sqrt{(a^2-1)}<1$.
Adding m to each member of the inequality $m<2P<m+1$.
Therefore, the integral part of m is odd.
(b) Let $[\sqrt{(a^2+1)}+a]^n=R+S\sqrt{(a^2+1)}=k$.
Then $[-\sqrt{(a^2+1)}+a]^n=R-S\sqrt{(a^2+1)}=\left(\frac{-1}{\sqrt{(a^2+1)}+a}\right)^n$.

If n is even, $0<R-S\sqrt{(a^2+1)}<1$. Adding k, $k<2R<k+1$. Whence the integral part of k is odd.